Do charged leptons oscillate?

Evgeny Kh. Akhmedov

Max-Planck-Institut für Kernphysik,
Postfach 10 39 80, 69029 Heidelberg, Germany, and
National Research Centre Kurchatov Institute,
123182 Moscow, Russia
E-mail: `akhmedov@mpi-hd.mpg.de`

ABSTRACT: The question of whether charged leptons oscillate is discussed in detail, with a special emphasis on the coherence properties of the charged lepton states created via weak interactions. This analysis allows one to clarify also an important issue of the theory of neutrino oscillations.

Contents

1. Introduction

Ever since the idea of neutrino oscillations was put forward [1, 2], the question of whether charged leptons can also undergo oscillations has been vividly discussed. While most of the authors conclude that such oscillations are not possible for one reason or another [3–9], others come to the opposite conclusion [10, 11].

Most of the arguments against the oscillations of charged leptons are based on the fact that mass eigenstates do not oscillate. This, however, does not answer the question of whether certain linear superpositions of charged leptons that could in principle be created through weak interactions would oscillate into different linear superpositions, leading to observable consequences. In the present note we address this question by examining the coherence properties of the charged lepton states produced in weak interactions. To the best of the present author's knowledge, this issue has not been previously studied in the literature. This discussion will also allow us to clarify an important issue of the theory of neutrino oscillations, namely: Why do neutrinos oscillate?

2. Do $e^{\pm}$, $\mu^{\pm}$ and $\tau^{\pm}$ oscillate into each other?

The answer to this question is the immediate 'no', the reason being that these charged leptons are mass eigenstates, i.e. states of definite mass. Let us review the simple arguments that show that such particles cannot undergo oscillations [3].

Assume first that at the time $t_0 = 0$ and position $\mathbf{x}_0 = 0$ a charged muon state is created:

$$|\Psi(0)\rangle = |\mu\rangle. \tag{2.1}$$

After time t and upon propagating the distance $\mathbf{x}$ this state evolves into[1]

$$|\Psi(t,\,\mathbf{x})\rangle \;=\; e^{-ip_\mu x}|\mu\rangle\,, \qquad p_\mu x \;=\; E_\mu t - \mathbf{p}_\mu\mathbf{x}\,, \tag{2.2}$$

where E_μ and $\mathbf{p}_\mu$ are the energy and 3-momentum of the muon, and for simplicity we have ignored the fact that muon is unstable (this is essentially irrelevant to the question we want to address). The probability for the muon to remain itself and not to oscillate into electron or tauon is then

$$P_{\mu\mu} \;=\; |\langle\mu|\Psi(t,\,\mathbf{x})\rangle|^2 \;=\; 1\,. \tag{2.3}$$

Consider now the situation where the initially produced charged lepton state is a linear superposition of e.g. muon and electron:

$$|\Psi(0)\rangle \;=\; \cos\theta|\mu\rangle \;+\; e^{i\alpha}\sin\theta|e\rangle \tag{2.4}$$

with real θ and α. The weights of μ and e in this state are $\cos^2\theta$ and $\sin^2\theta$ respectively. The evolved state is then

$$|\Psi(t,\,\mathbf{x})\rangle \;=\; e^{-ip_\mu x}\cos\theta|\mu\rangle \;+\; e^{-ip_e x}e^{i\alpha}\sin\theta|e\rangle\,. \tag{2.5}$$

The probabilities of finding μ and e in the evolved state are

$$P_\mu = |\langle\mu|\Psi(t,\,\mathbf{x})\rangle|^2 \;=\; |e^{-ip_\mu x}\cos\theta|^2 \;=\; \cos^2\theta\,, \tag{2.6}$$

$$P_e = |\langle e|\Psi(t,\,\mathbf{x})\rangle|^2 \;=\; |e^{-ip_e x+i\alpha}\sin\theta|^2 \;=\; \sin^2\theta\,, \tag{2.7}$$

i.e. the same as in the initial state (2.4). Thus, there are no oscillations between mass-eigenstate charged leptons e, μ and τ, no matter if the initial state is pure or a coherently mixed one. The reason for this is that mass eigenstates evolve by simply picking up phase factors whose moduli are always equal to unity.

It should be noted that the same argument applies to neutrinos: an initially produced flavor state, say ν_e, can oscillate with some probability into ν_μ or ν_τ, but the weights of the mass eigenstates ν_1, ν_2 and ν_3 in such a state will not change with time.

3. Oscillation between superpositions of e, μ and τ

A natural question then is: Can we imagine a situation when one creates a coherent superposition of e, μ and τ and then also <u>detects</u> their coherent superposition (the same or different) rather than individual mass-eigenstate charged leptons? If this were possible, one would be able to observe oscillations between such mixed charged lepton states [3].

Closely related to the above question is the following one: Why do we say that in charged-current weak interactions charged leptons are emitted and detected as mass eigenstates and neutrinos as flavor states (superpositions of mass eigenstates) and not vice versa?

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[1]For simplicity, in this section we confine our discussion to the plane wave approximation. It is easy to see that a more rigorous consideration in terms of wave packets would yield the same result, the reason being that the wave packets are normalized.

Or not both as some superpositions of mass eigenstates? After all, charged-current weak interactions are completely symmetric with respect to neutrinos and charged leptons,

$$\mathcal{L}_{\text{CC}} = -\frac{g}{\sqrt{2}} \left(\bar{e}_{aL} \gamma^\mu U_{ai} \nu_{iL} \right) W_\mu^- \; + \; h.c. \,, \qquad (a = e, \mu, \tau, \quad i = 1, 2, 3) \,, \qquad (3.1)$$

with the leptonic mixing matrix U coming from the diagonalization of the mass matrices of both charged leptons and neutrinos, so why cannot charged leptons be created and absorbed in weak interactions as coherent superpositions of mass eigenstates? What is the origin of the disparity between neutrinos and charged leptons?

One might suspect that this disparity comes about because of the enormous difference between the masses of charged leptons and neutrinos, and as we shall see, this is indeed the case. However, it is important to understand how exactly this mass difference comes into play.

Let us consider the problem in more detail. The question we want to address is how do we know that a charged lepton emitted or absorbed in a weak interaction process is either e or μ or τ but not their coherent superposition. This actually amounts to asking why neutrinos oscillate, because it is the fact that charged leptons participate in weak interactions as mass eigenstates that "measures" the neutrino flavor, i.e. ensures that neutrinos are emitted and captured as well-defined coherent superpositions of mass eigenstates.[2]

In the case of nuclear β decay the situation is simple: only $e^\pm$ can be emitted together with a neutrino or antineutrino, because there is no energy available to produce $\mu^\pm$ or $\tau^\pm$. The same is also true for muon decays $\mu^\pm \to e^\pm \nu \bar{\nu}$. Thus, in these cases the emitted charged lepton is obviously a pure mass eigenstate.

Consider, however, decays of charged pions $\pi^\pm \to l^\pm \nu$ (or similarly for charged kaons). Here the decay energy is sufficient for the production of both electrons and muons, i.e. $l = e, \mu$. So how do we know that the produced charged lepton is either e or μ and not their coherent superposition? As was already pointed out, this is actually the same as asking how do we know that the emitted neutrino is either ν_e or ν_μ. Of course, if e.g. a μ^+ produced in the pion decay is detected, than we know that the neutrino born in the same process is ν_μ. But what if the charged lepton is not detected, as it is usually the case?

To illustrate the arising problem, consider a hypothetical situation when neutrinos produced or absorbed in weak interactions are mass eigenstates ν_1, ν_2 and ν_3, whereas the associated charged leptons are

$$
\begin{aligned}
|e_1\rangle &= U_{1e}|e\rangle \; + \; U_{1\mu}|\mu\rangle \; + \; U_{1\tau}|\tau\rangle \,, \\
|e_2\rangle &= U_{2e}|e\rangle \; + \; U_{2\mu}|\mu\rangle \; + \; U_{2\tau}|\tau\rangle \,, \\
|e_3\rangle &= U_{3e}|e\rangle \; + \; U_{3\mu}|\mu\rangle \; + \; U_{3\tau}|\tau\rangle \,,
\end{aligned}
\qquad (3.2)
$$

which are emitted or detected together with ν_1, ν_2 and ν_3 respectively. This possibility is perfectly consistent with the charged-current interaction Lagrangian (3.1). However, if this were the case, then charged leptons e_1, e_2 and e_3 would oscillate into each other, while

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[2]Note that for charged leptons their flavor is defined to coincide with their mass.

neutrinos would not be able to oscillate. We know that in reality neutrinos do oscillate, so what is wrong with this apparently consistent possibility?

To make the problem look even worse, one could conceive a situation in which both charged leptons and neutrinos participating in charged-current weak interactions are coherent superpositions of their respective mass eigenstates:

$$|e_\beta\rangle = \sum_a W^*_{\beta a}|e_a\rangle\,, \qquad |\nu_\beta\rangle = \sum_i V^*_{\beta i}|\nu_i\rangle\,, \quad e_a = e, \mu, \tau, \quad i = 1, 2, 3, \qquad (3.3)$$

where W and V are 3×3 unitary matrices satisfying the condition

$$W^\dagger V = U \qquad (3.4)$$

but otherwise arbitrary. Eq. (3.3) defines the new quantum number of neutrinos and charged leptons which we shall call "odor" to distinguish it from the usual leptonic flavor. The special case $W = \mathbb{1}$, $V = U$ corresponds to the standard situation where the charged leptons participating in weak interactions are mass eigenstates, while neutrinos are the flavor eigenstates ν_e, ν_μ and ν_τ, whereas the special case $W = U^\dagger$, $V = \mathbb{1}$ corresponds to the situation where the weak-eigenstate charged leptons are given by eq. (3.3), and neutrinos are emitted and absorbed as mass eigenstates.

Had weak interactions selected the neutrino states ν_β defined in eq. (3.3) as weak eigenstates, then by detecting such a neutrino we would measure the odor of the associated charged lepton; in this case the charged leptons states e_β could oscillate into each other. However, these oscillations would only occur if both neutrino and charged lepton produced in the same decay were detected, i.e. they would be a manifestation of an EPR-like correlation [12]. Likewise, for neutrinos to oscillate, one would have to measure their odor by detecting the charged lepton state emitted in the same decay. At the same time, neutrinos are known to oscillate even when the associated charged leptons are not detected. To understand why this happens and why charged leptons do not oscillate we have to study the coherence properties of the charged lepton states produced in weak interaction processes.

4. Coherence properties of charged lepton states

Unlike neutrinos which can be produced or detected only via weak interactions,[3] charged leptons participate also in electromagnetic interactions and are usually detected through them. The electromagnetic interactions are, however, flavor-blind, and therefore of no interest to us here. We shall thus concentrate on the coherence properties of charged lepton states produced or detected in weak-interaction processes.

The energy E and momentum p of a particle produced, e.g., in some decay process have quantum-mechanical uncertainties, σ_E and σ_p. This, in particular, means that the particle should be described by a wave packet of the spatial size $\sigma_x \sim 1/\sigma_p$ rather than by a plane wave. The knowledge of the particle's energy and momentum and their corresponding uncertainties would allow one to determine the squared mass of the particle with an uncertainty σ_{m^2}.

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[3]Ignoring possible new interactions responsible for the neutrino mass generation.

Let us consider for definiteness $\pi^\pm \to l^\pm \nu$ decays. The energies and momenta of the produced charged leptons and their corresponding quantum-mechanical uncertainties are determined by the decay conditions. If the uncertainty in the inferred mass of the charged lepton σ_{m^2} satisfies[4]

$$\sigma_{m^2} < m_\mu^2 - m_e^2 \,, \tag{4.1}$$

then for for each decay event one would exactly know which particular charged lepton was produced. The fact that in each decay either μ or e is produced (with their respective probabilities) would imply that the produced charged lepton state is an incoherent mixture of μ and e. If, on the contrary,

$$\sigma_{m^2} > m_\mu^2 - m_e^2 \,, \tag{4.2}$$

then it will be in principle impossible to determine which mass-eigenstate charged lepton was produced in the decay process. The amplitudes of the emission of μ and e would then add coherently, i.e. the emitted charged lepton state would be a coherent superposition of μ and e. The situation here is quite similar to that with the electron interference in double slit experiments: If there is no way to find out which slit the detected electron has passed through, the detection probability will exhibit an interference pattern, but if such a determination is possible, the interference pattern will be washed out.

Now let us estimate the mass uncertainties of charged leptons produced in $\pi^\pm \to l^\pm \nu$ decays. Assuming that the uncertainties σ_E and σ_p are uncorrelated, from the relativistic relation between the mass, energy and momentum of a free particle $m^2 = E^2 - p^2$ where $p \equiv |\mathbf{p}|$, one finds

$$\sigma_{m^2} = \left[(2E\sigma_E)^2 + (2p\sigma_p)^2 \right]^{1/2} \,. \tag{4.3}$$

For an isolated decaying particle (or when its interaction with the environment can be neglected) the quantum-mechanical uncertainties of the energies of the decay products are essentially given by their parent's decay width. Thus, for charged leptons produced in $\pi^\pm \to l^\pm \nu$ decays we have

$$\sigma_E \simeq \Gamma_\pi = \Gamma_\pi^0 / \gamma \,, \tag{4.4}$$

where $\gamma = (1 - v^2)^{-1/2}$ is the pion's Lorentz factor and

$$\Gamma_\pi^0 = 2.5 \cdot 10^{-8} \text{ eV} \tag{4.5}$$

is its rest-frame decay width.

The uncertainty in the momentum of a particle produced in the decay is approximately given by the reciprocal of its coordinate uncertainty σ_x, which is essentially its velocity times the lifetime of the parent particle. Thus, for charged leptons produced in pion decay

$$\sigma_p \simeq [(p/E)\tau_\pi]^{-1} = (E/p)\Gamma_\pi \,. \tag{4.6}$$

From eq. (4.4) it then follows that the two terms in the square brackets in eq. (4.3) are approximately equal, and one finally gets

$$\sigma_{m^2} \simeq 2\sqrt{2}\, E\sigma_E \,. \tag{4.7}$$

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[4]Our arguments here are similar to those used in the discussion of the coherence of neutrinos in [13].

It should be noted that σ_{m^2}, being the uncertainty of a Lorentz-invariant quantity, must itself be Lorentz invariant. Our estimate (4.7) satisfies this condition. Indeed, when going from the pion's rest frame to the laboratory frame, the energies of the emitted leptons averaged over the directions of their rest-frame velocities with respect to the pion boost direction scale as $E \to \gamma E$. Together with eq. (4.4), this proves Lorentz invariance of (4.7).

The uncertainty in the charged-lepton mass determination in pion decay can therefore be estimated in pion's rest frame:

$$\sigma_{m^2} \simeq 2\sqrt{2}\,\bar{E}\,\Gamma_\pi^0 \simeq 2\sqrt{2} \cdot 90\ \text{MeV} \cdot 2.5 \cdot 10^{-8}\ \text{eV} \simeq 6.4\ \text{eV}^2\,, \tag{4.8}$$

where $\bar{E} \simeq 90\,\text{MeV}$ is the average energy of charged leptons produced in pion decays at rest. This has to be compared with $m_\mu^2 - m_e^2 \simeq (106\ \text{MeV})^2$; obviously, the condition in eq. (4.1) is satisfied with a huge margin, which means that different mass-eigenstate charged leptons are emitted incoherently. Very similar estimates apply also to the decays of charged kaons.

Thus, we conclude that charged leptons born in the decays of pions or kaons (as well as in nuclear beta decays and in muon decays) are always emitted as mass eigenstates and not as coherent superpositions of different mass eigenstates because of their very large mass squared differences. Therefore even oscillations between the states e_1, e_1 and e_3 (or between different odor states e_β) discussed in the previous section are not possible — these states just are not produced. Similar considerations apply to the absorption of charged leptons in weak-interaction processes.[5]

Does this conclusion hold for all conceivable weak processes? The energies and momenta of charged leptons produced in pion and kaon decays are relatively small because of the small mass of the decaying particle, which implies small phase-space volumes available for the decay products. This (together with the chiral suppression which requires that the decay amplitudes be proportional to the lepton's mass) also explains relative smallness of the pion and kaon decay widths, which determine the uncertainties of the energies and momenta of the produced charged leptons. Altogether this results in rather small uncertainties σ_{m^2} of the lepton masses and ensures that the condition (4.1) is satisfied.

Let us now consider W-boson decays $W^\pm \to l_a^\pm \nu$ ($l_a = e,\ \mu,\ \tau$), which are characterized by large phase-space volumes and also do not suffer from the chiral suppression. The partial decay widths for such decays are

$$\Gamma^0_{W \to l_a \nu} \simeq \frac{G_F m_W^3}{6\sqrt{2}\pi} \simeq 230\ \text{MeV}\,, \tag{4.9}$$

where G_F is the Fermi constant and $m_W \simeq 80.4\,\text{GeV}$ is the W-boson mass. For W decay at rest we therefore have the following estimate for the uncertainty of the charged lepton

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[5]If σ_{pP} and σ_{pD} are the momentum uncertainties associated with the production and detection processes, then eq. (4.3) still holds with σ_p being the effective uncertainty which is found from the relation $\sigma_p^{-2} = \sigma_{pP}^{-2} + \sigma_{pD}^{-2}$, i.e. σ_p is dominated by the smallest of the two uncertainties. The same applies to the energy uncertainties. These results can be obtained from the wave packet treatment of the production and detection processes and are similar to the corresponding results for neutrinos (see, e.g., [14, 15]).

mass:[6]

$$\sigma_{m^2} \sim 2\sqrt{2}\, E\sigma_E \simeq 2\sqrt{2} \cdot 40 \text{ GeV} \cdot 230 \text{ MeV} \simeq (5 \text{ GeV})^2. \tag{4.10}$$

Thus, we have

$$\sigma_{m^2} \gg m_\mu^2 - m_e^2, \qquad \sigma_{m^2} > m_\tau^2 - m_\mu^2 \simeq (1.77 \text{ GeV})^2, \tag{4.11}$$

which means that all three charged leptons are produced coherently in $W^\pm$ decays. Since σ_{m^2} is a Lorentz-invariant quantity, the same estimates (4.10), (4.11) and the same conclusion apply also for $W^\pm$ decays in flight. Thus, charged leptons are produced in $W^\pm \to l^\pm \nu$ decays as coherent superpositions of e, μ and τ.

Does this mean that one can observe oscillations of such charged leptons if the detection process is also coherent? For the observability of the charged lepton oscillations it is not sufficient that the lepton state be produced as a coherent superposition of mass eigenstates; the emitted state should also preserve its coherence until it is detected. The coherence loss can occur for mixed states because of the finite effective spatial size σ_x of the wave packet describing the propagating state. Since different mass-eigenstate components of the mixed state propagate with different group velocities $v_g = \partial E/\partial p$, after the coherence time

$$t_{\text{coh}} \simeq \frac{\sigma_x}{\Delta v_g} \tag{4.12}$$

the wave packets describing the individual mass eigenstates separate and can no longer interfere, which means that the state loses its coherence.

Let us now estimate the coherence time for the charged lepton states produced in $W^\pm \to l^\pm \nu$ decays at rest and the corresponding coherence length x_{coh} (which for relativistic leptons coincides with the coherence time). The maximum coherence length corresponds to the minimum group velocity difference,

$$(\Delta v_g)_{\text{min}} = \frac{p_e}{E_e} - \frac{p_\mu}{E_\mu} \simeq 2\frac{m_\mu^2 - m_e^2}{m_W^2}. \tag{4.13}$$

For the maximum coherence length we therefore find from eqs. (4.12), (4.9) and (4.13)

$$(x_{\text{coh}})_{\text{max}} \simeq [\Gamma^0_{W \to l_a \nu}(\Delta v_g)_{\text{min}}]^{-1} \simeq \frac{3\sqrt{2}\pi}{G_F m_W (m_\mu^2 - m_e^2)} \simeq 2.5 \times 10^{-8} \text{ cm}. \tag{4.14}$$

Thus, even though charged leptons are emitted in $W^\pm \to l^\pm \nu$ decays as coherent superpositions of mass eigenstates, they lose their coherence upon propagating only $\sim 10^{-8}$ cm from their birthplace, i.e. over interatomic distances. This means that coherent effects in the $l^\pm$ production are unobservable, and for all practical purposes one can consider the charged leptons produced in $W^\pm \to l^\pm \nu$ decays at rest to be an incoherent mixture of e, μ and τ.

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[6]One might argue that the total W-boson width Γ^0_W rather than the partial widths of $W^\pm \to l_a^\pm \nu$ decays should be used in this estimate. This would increase the estimate in eq. (4.10) by about a factor of 10, strengthening our conclusions.

What about $W^{\pm} \to l^{\pm}\nu$ decays in flight? Let γ be the Lorentz factor of $W^{\pm}$. The minimum group velocity difference of the produced charged leptons $(\Delta v_g)_{\min} \simeq \Delta m_{\mu e}^2/2E^2 \equiv (m_{\mu}^2 - m_e^2)/2E^2$ and the partial decay width of $W^{\pm}$ scale with γ as

$$(\Delta v_g)_{\min} \to \gamma^{-2}(\Delta v_g)_{\min} , \qquad \Gamma_{W \to l_a \nu}^0 \to \gamma^{-1}\Gamma_{W \to l_a \nu}^0 . \qquad (4.15)$$

Therefore the maximum coherence length scales as

$$(x_{\mathrm{coh}})_{\max} \to \gamma^3 (x_{\mathrm{coh}})_{\max} . \qquad (4.16)$$

In order for $(x_{\mathrm{coh}})_{\max}$ to be, say, larger than $1\,m$, one would need $\gamma \gtrsim 1600$, or $E_W \gtrsim 130\,\mathrm{TeV}$, which is far above presently feasible energies.

It is easy to see that the condition of having a coherent emission of charged leptons in a decay process and the condition that the leptons keep their coherence over a macroscopic distance L tend to put conflicting constraints on the size σ_x of the charged leptons' wave packet. Indeed, as follows from eqs. (4.2) and (4.7), the first condition requires

$$\sigma_x \sim \sigma_p^{-1} \simeq \sigma_E^{-1} < (\Delta m_{\mu e}^2/2\sqrt{2}E)^{-1} , \qquad (4.17)$$

whereas the second one yields

$$\sigma_x > (\Delta v_g)_{\min}L \simeq (\Delta m_{\mu e}^2/2E^2)\,L , \qquad (4.18)$$

in accordance with eqs. (4.12) and (4.13). To reconcile the upper and lower limits on σ_x given in eqs. (4.17) and (4.18), L must satisfy

$$L < \frac{4\sqrt{2}\,E^3}{(\Delta m_{\mu e}^2)^2} \;\simeq\; 8.9 \times 10^{-10} \left(\frac{E}{\mathrm{GeV}}\right)^3 \,\mathrm{cm} . \qquad (4.19)$$

Note that this condition is independent of the size of the wave packet σ_x and therefore of the decay width of the parent particle. From (4.19) it follows that in order for charged leptons to be born coherently and keep their coherence over a distance $L \gtrsim 1\,m$ they should have energies greater than at least $4.8\,\mathrm{TeV}$. A condition similar to that in eq. (4.19) exists also for neutrino oscillations, but in that case it is much easier to satisfy because of the smallness of neutrino mass squared differences. In particular, for a baseline $L \gtrsim 1\,km$ one would only need neutrino energies $E_{\nu} \gtrsim 20\,\mathrm{eV}$.

It should be stressed that the condition (4.19) (and the similar condition for neutrinos) is necessary but in general not sufficient for a mixed state to be coherently produced and maintain its coherence over the distance L: it only ensures the consistency of the conditions (4.17) and (4.18) but not their separate fulfilment.

Several comments are in order. First, a sufficiently coherent detection can improve the overall coherence of the total lepton production — propagation — detection process [16]. In particular, even if the wave packets have already separated, they can still overlap and interfere in the detector if their separation is not too large and if the detection process is sufficiently coherent (i.e., lasts a sufficiently long time). In that case the separated wave packets arrive at the detector before the detection process is over. The coherence length

is therefore the distance over which the wave packets separate to such an extent that they can no longer overlap in the detector. Our discussion of the separation of wave packets earlier in this section still holds if one understands by σ_x the effective wave packet size, $\sigma_x = \sigma_p^{-1} \equiv (\sigma_{pP}^{-2} + \sigma_{pD}^{-2})^{1/2}$, which takes the detection process into account (see footnote 5). In our numerical estimates we were assuming that the coherence of the detection process is not too different from that of the production process.

Second, in our discussion of the loss of coherence caused by the wave packet separation we have assumed that the size of the wave packet does not change with time, i.e. neglected the wave packet spreading. Such a spreading in general occurs when the group velocity depends on the particle's momentum (i.e. in the presence of dispersion). This is, in particular, the case for free relativistic massive particles, for which $\partial v_g/\partial p = m^2/E^3$. The asymptotic (large-$t$) spreading velocity is then $v_\infty = m^2/(E^3\sigma_x)$.[7] The spreading increases the spatial size of the wave packets and therefore tends to counter the effect of the wave packet separation. The coherence can be recovered at large enough times provided that the asymptotic spreading velocity is larger than the difference of the group velocities:

$$v_\infty = \frac{m^2}{E^3\sigma_x} > \frac{|\Delta m_{ab}^2|}{2E^2} \qquad (a, b = e,\,\mu,\,\tau)\,. \tag{4.20}$$

From $\sigma_x^{-1} \sim \sigma_p \simeq \Gamma$ and the fact that $|\Delta m_{ab}^2| \simeq \max(m_a^2,\, m_b^2)$ it follows that the condition (4.20) reduces to the following inequality between the decay width of the parent particle and the energy of the produced charged lepton state:

$$\Gamma \gtrsim E/2\,. \tag{4.21}$$

In reality this condition is never satisfied, which justifies our neglect of the wave packet spreading.

We have found that the charged lepton states born in the $W^\pm$ decays are produced coherently and can maintain their coherence up to macroscopic distances provided that $E_W \gtrsim 100\,\text{TeV}$. However, as follows from the discussion in section 3, for these coherence effects to be experimentally observable the following two conditions have to be satisfied: (i) at the production, an accompanying neutrino must be detected, thus providing a measurement of the composition of the emitted charged lepton state. Moreover, this neutrino must not be a flavor eigenstate ν_e, ν_μ or ν_τ (otherwise the flavor of the produced charged lepton would be measured, so that it would be either e or μ or τ but not their coherent superposition); (ii) the detection process should be able to discriminate between different coherent superpositions of charged leptons. Obviously, the standard charged-current weak interactions cannot meet these two conditions: the absorption of a neutrino state different from a flavor eigenstate would be accompanied by the emission of a mixed charged-lepton state, which would again have to be identified by its charged-current interaction, leading to the emission of the same mixed neutrino state. To break the circle, new interactions are necessary, and therefore we turn to possible new physics effects now.

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[7]This expression can be readily obtained from the general formulas given in section 3 of [17].

5. New physics?

Assume very heavy sterile neutrinos N_i exist (as required, e.g., by the seesaw mechanism of neutrino mass generation) and consider their decay into a charged lepton and charged Higgs boson:

$$N_i \to e_i^- + \Phi^+ . \tag{5.1}$$

This would also require the existence of an extra Higgs boson doublet because the charged component of the standard model Higgs is eaten up by the $W^\pm$ bosons through the Higgs mechanism .[8] The decays in eq. (5.1) are caused by the Yukawa coupling Lagrangian

$$\mathcal{L}_Y = Y_{ai} \bar{L}_a N_{Ri} \Phi \ + \ h.c. , \tag{5.2}$$

where $L_a = (\nu_{La}, e_{La})^T$ are the SU(2)$_L$ doublets of the left handed lepton fields. We work in the basis where the mass matrices of heavy sterile neutrinos and charged leptons have been diagonalized; the Yukawa coupling matrix Y_{ai} is in general not diagonal in this basis, so that in the decay of a mass-eigenstate sterile neutrino N_i any of the three charged leptons $e_a = e, \mu, \tau$ can be produced. We want to find out under what conditions the produced charged lepton state e_i in eq. (5.1) is a coherent superposition of the mass eigenstates e_a, in which case it is given by

$$|e_i\rangle = [(Y^\dagger Y)_{ii}]^{-1/2} \sum_a Y_{ia}^\dagger |e_a\rangle , \tag{5.3}$$

and how long this state can maintain its coherence.

Neglecting the Higgs boson and charged lepton masses compared to the mass of the sterile neutrino M_i, for the rest-frame decay width of N_i we find

$$\Gamma_i^0 \simeq \alpha_i M_i , \qquad \text{where} \qquad \alpha_i \equiv \frac{(Y^\dagger Y)_{ii}}{16\pi} . \tag{5.4}$$

We now apply the arguments of the previous section to the decay (5.1). The condition (4.2) which has to be satisfied in order for the charged lepton state to be produced as a coherent superposition of e, μ and τ reads

$$2\sqrt{2} \, E \, \Gamma_i^0 \simeq 2\sqrt{2} \, (M_i/2) \, \alpha_i M_i > \max\{m_\mu^2 - m_e^2, \ m_\tau^2 - m_\mu^2\} , \tag{5.5}$$

or

$$\alpha_i > 2.2 \, (M_i/\text{GeV})^{-2} . \tag{5.6}$$

From eq. (4.12) we find the coherence length for the emitted charged lepton state:

$$x_{\text{coh}} \simeq \frac{M_i^2}{2\Gamma_i^0(m_\tau^2 - m_\mu^2)} \simeq 3.1 \times 10^{-15} \, \alpha_i^{-1} \frac{M_i}{\text{GeV}} \ \text{cm} . \tag{5.7}$$

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[8]An alternative possibility, to have the decay (5.1) in the early universe above the electroweak symmetry breaking temperature, is of no interest to us: even though the charged component of the standard model Higgs would be physical in that case, the charged leptons would be massless and so would not oscillate.

From eq. (5.6) it then follows that

$$x_{\text{coh}} < 1.4 \times 10^{-15} \text{ cm } (M_i/\text{GeV})^3 \,. \qquad (5.8)$$

Thus, for the charged lepton state to maintain its coherence over the distance of $\sim 1\,m$, the sterile neutrino must have the mass $M_i \gtrsim 400\,\text{TeV}$. Eq. (5.6) then implies that the Yukawa couplings Y_{ij} must satisfy $(Y^\dagger Y)_{ii} \gtrsim 1.3 \times 10^{-11}$. If only e and μ are to be produced coherently, a significantly milder lower limit on the sterile neutrino mass results: $M_i \gtrsim 10\,\text{TeV}$, whereas for the Yukawa couplings one gets the constraint $(Y^\dagger Y)_{ii} \gtrsim 8.5 \times 10^{-11}$. Note that for N_i decay in flight the right hand side of eq. (5.8) has to be multiplied by γ^3, which amounts to replacing the factor $(M_i/\text{GeV})^3$ there by $(E_i/\text{GeV})^3$. Thus, for N_i decays in flight the condition of macroscopic coherence length puts a lower bound only on the energy of the sterile neutrinos, so that they can be relatively light.

If the condition (5.6) for the coherent creation of the charged lepton state in the decay (5.1) is satisfied and this state is detected through the inverse decay process before it loses its coherence, it may exhibit oscillations: a mass eigenstate sterile neutrino N_j different from N_i can be produced in the detection process, meaning that the state e_i has oscillated into e_j. The measurement of the "flavor" of the originally produced e_i, i.e. of its composition with respect to the mass eigenstates e, μ and τ (as given in eq. (5.3)), is provided by the fact that the decaying sterile neutrino is a mass eigenstate.

In our discussion of the decay (5.1) we were assuming that the sterile neutrinos N_i are heavier than the charged Higgs boson Φ. If Φ is heavier than N_i, then decays

$$\Phi^\pm \to N_i + e_i^\pm \qquad (5.9)$$

are possible. This case can be analyzed quite analogously. Sterile neutrinos, being very heavy, are either emitted incoherently or lose their coherence almost immediately, providing a measurement of the "flavor" of the charged lepton state. Charged leptons then would be able to oscillate, leading to a non-zero probability of the emission or absorption of a different sterile neutrino mass eigenstate N_j in the processes $e_j^\pm + \Phi^\mp \to N_j$ or $e_j^\pm + N_j \to \Phi^\pm$. Thus, in the cases of decays (5.1) and (5.9) we have the roles of neutrinos and charged leptons reversed as compared to the usual situation because of sterile neutrinos being much heavier than the charged leptons.

6. Charged lepton oscillation lengths and averaging over the source/detector size

Up to now in our discussion we have been only considering the coherence properties of individual charged lepton states produced in decays of a single particle. However, in real experiments one normally has to deal with beams originating from the decays of the parent particles confined within a certain source volume, and the coordinate of the production point is usually only known with an uncertainty of the order of the (macroscopic) size of the source L_S. Likewise, the coordinate of the detection point is only known with the uncertainty of the order of the detector size L_D. In calculating the event rates one

has to integrate over the coordinates of the production and detection points within their respective allowed spatial regions. Thus, the effective uncertainties of the coordinates of the production and detection points of charged leptons are usually much larger than the corresponding intrinsic quantum-mechanical uncertainties. As we shall see, because of this the requirement of macroscopic coherence lengths of charged leptons puts a very stringent lower bound on their energies. This bound stems from the condition of no averaging of the charged lepton oscillations over the lengths of the source and detector and is actually more stringent than the one coming from the condition of no wave packet separation.

If the charged lepton oscillation length l_{osc} is much smaller than the size of the source in the direction of the beam L_S, then the integration over the production point would average out the interference terms in the squared modulus of the amplitude of the process. The same is also true for the integration over the detection point provided that $l_{\text{osc}} \ll L_D$. The absence of the interference terms would mean that the coherence effects in the charged lepton states are unobservable, and in each event a certain mass-eigenstate charged lepton is emitted or absorbed with its respective probability.

Let us now estimate the energies E_0 of the decaying parent particle that are necessary for l_{osc} to take macroscopic values. The maximum oscillation length (corresponding to the smallest mass squared difference) is

$$(l_{\text{osc}})_{\max} = \frac{2\pi}{|E_\mu - E_e|} \simeq 2.5 \ m \ \frac{[(E_0/2)\,(\text{MeV})]}{\Delta m_{\mu e}^2 \,(\text{eV}^2)} \simeq 1.1 \times 10^{-11}(E_0/\text{GeV}) \ cm\,. \quad (6.1)$$

Therefore in order to have, e.g., $l_{\text{osc}} \gtrsim 1\,m$ one would need

$$E_0 \gtrsim 9 \times 10^{12} \ \text{GeV}\,, \quad (6.2)$$

which is far above the experimentally accessible energies (except, probably, for the highest-energy cosmic rays).

7. Discussion and conclusions

We have studied the coherence properties of the charged lepton states produced in weak-interaction processes and demonstrated that in those cases when the production of more than one type of mass-eigenstate charged leptons is kinematically allowed, the charged lepton states are either produced as incoherent mixtures of e, μ and τ, or they lose their coherence over microscopic distances, except at extremely high energies, not accessible to present experiments. The reason for this difference between the coherence properties of neutrinos and charged leptons produced in weak-interaction processes is the enormous disparity between the masses of these two leptonic sectors of the standard model.

We have also discussed charged lepton production in decays of heavy sterile neutrinos N_i and demonstrated that in that case the oscillations between different coherent superpositions of e, μ and τ are possible, leading to potentially observable effects. The conditions for the observability of the oscillations of charged leptons produced in N_i decays have been identified.

We have studied three sources of decoherence of the charged lepton states: (i) lack of coherence at production; (ii) loss of coherence due to the wave packet separation, and (iii) washout of coherence due to the averaging over the source and/or detector size. We have not considered the effects of μ and τ decays which could also cause loss of coherence of the charged lepton states. Except at extremely high energies, these decays occur on longer length scales than the decoherence due to the wave packet separation. For example, in the case of $W^{\pm}$ decays, the decay length of the produced $\tau^{\pm}$ becomes shorter than the coherence length due to the wave packet separation $x_{\rm coh}$ only for $E_W \gtrsim 2 \times 10^6$ GeV, whereas the $\mu^{\pm}$ decay length becomes shorter than the corresponding $x_{\rm coh}$ only for $E_W \gtrsim 3 \times 10^9$ GeV. For the charged leptons produced in the decays of heavy sterile neutrinos N_i the instability of τ and μ becomes relevant for $E_i \gtrsim 1.3 \times 10^6$ GeV and $E_i \gtrsim 1.5 \times 10^{10}$ GeV respectively.

A necessary condition for the observability of the charged lepton oscillations is that they be emitted and detected as nontrivial linear superpositions of the mass eigenstates e, μ and τ. For charged leptons produced in charged-current weak interactions the required "measurement" of the composition of their state can be achieved if, e.g, the accompanying neutrino is detected as a mass eigenstate. The measurements of the neutrino mass could in principle be performed through the time-of-flight techniques or through observation of decays of heavier neutrinos into lighter ones.[9] However, it is easy to see that the current limits on neutrino masses and neutrino instability imply that the baselines (and flight times) necessary for such measurements are extremely large; this means that even if such measurements are performed, by the time they are done charged leptons will have already lost their coherence.

The fact that charged leptons are always born in charged-current weak interactions as incoherent mass eigenstates or lose their coherence practically immediately has important consequences for neutrino oscillations. For neutrinos to oscillate, they should be produced and detected as well-defined coherent superpositions of mass eigenstates. This is trivially satisfied for neutrinos from β decay, in which only electron-flavor neutrinos or antineutrinos are produced because the only charged leptons which can be emitted are $e^{\pm}$. The same is true for electron neutrinos or antineutrinos from $\mu \to e\nu\bar{\nu}$ decays, whereas the flavor of the other neutrino emitted in the same process is measured by the fact that the decaying particle (muon) is a mass eigenstate. However, in the decays such as $\pi^{\pm} \to l^{\pm}\nu$, $K^{\pm} \to l^{\pm}\nu$ or $W^{\pm} \to l^{\pm}\nu$ the production of more than one charged lepton species is kinematically allowed. It is the lack of coherence of the produced charged lepton state or the loss of its coherence over microscopic distances that ensures that in each decay event a particular mass-eigenstate charged lepton is emitted and thus provides a measurement of the flavor of the associated neutrino. Only for this reason neutrinos emitted in such processes oscillate even when the associated charged lepton is not detected.

Let us now briefly summarize our main conclusions:

- Charged leptons e, μ and τ do not oscillate into each other because they are mass eigenstates. Since in β decays and muon decays the production of $\mu^{\pm}$ and $\tau^{\pm}$ is

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[9]The author is grateful to J. Rich for raising these points.

kinematically forbidden, the are no charged lepton oscillations associated with these processes.

- Charged leptons born in $\pi^\pm$ and $K^\pm$ decays are produced incoherently, i.e. are either $\mu^\pm$ or $e^\pm$, but not their linear superpositions. Therefore they do not oscillate.

- For charged leptons produced in $W^\pm$ decays the coherence production condition is satisfied. However, for $W^\pm$ decays at rest the coherence is lost over microscopic distances because of the wave packet separation. For decays in flight with $E_W \gtrsim 100\,\mathrm{TeV}$ the coherence lengths can formally take macroscopic values; yet, the coherence effects in the charged lepton sector are unobservable even in this case because the standard charged-current weak interactions cannot provide a measurement of the composition of the initially produced as well as of the evolved charged lepton state.

- Charged lepton states produced in the decays of heavy sterile neutrinos can be coherent superpositions of e, μ and τ. They can maintain their coherence over macroscopic distances provided that their energies exceed a few hundred TeV. Such charged lepton states could oscillate, and their oscillations could lead to observable consequences.

- Integration over the macroscopic sizes of the source and detector would wash out the effects of the oscillations of charged leptons unless the corresponding oscillation length exceeds the source and detector sizes in the direction of the beam, L_S and L_D. The requirement of no washout for $L_S, L_D \gtrsim 1\,m$ puts a stringent lower bound on the energy of the decaying parent particle: $E_0 \gtrsim 10^{13}\,\mathrm{GeV}$.

- Neutrinos produced in the processes in which the emission of more than one species of charged leptons is kinematically allowed oscillate even if the associated charged lepton is not detected, because the measurement of their flavor is provided by the decoherence of the associated charged lepton state.

Thus, the short answer to the question raised in the title of this paper is 'no', at least if no new physics is involved. But even if the relevant new physics exists, an observation of the oscillations between different coherent superpositions of e, μ and τ would probably require extremely high energies, not accessible to current and most likely also to future experiments.

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